DISTRIBUTED OUTPUT FEEDBACK INDIRECT MRAC OF CONTINUOUS-TIME MULTI-AGENT LINEAR SYSTEMS*

JIAN GUO†, YANJUN ZHANG‡, AND JI-FENG ZHANG §

Abstract. This paper studies the distributed leader-follower output consensus problem for continuous-time uncertain multi-agent linear systems in general input-output forms. Specifically, we extend the well-known output feedback indirect model reference adaptive control (MRAC) and develop a fully distributed output feedback indirect MRAC scheme to achieve closed-loop stability and asymptotic leader-follower output consensus. Compared with the existing results, the proposed distributed MRAC scheme has the following characteristics. First, the orders of each agent's pole/zero polynomials, including the followers and the leader, can differ from others, and the parameters in each follower's pole/zero polynomials are unknown. Second, the proposed adaptive control law of each follower solely relies on the local input and output information without requiring the state observer and the structural matching condition on the followers' dynamics, commonly used in the literature. Third, for any given leader with a relative degree n^*, the leader-follower output tracking error and its derivatives up to the n^*-th order converge to zero asymptotically, which has never been reported in the literature. Finally, a simulation example verifies the validity of the proposed distributed MRAC scheme.

Key word. Model reference adaptive control, distributed output feedback, multi-agent systems, leader-follower consensus

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1. Introduction.

Multi-agent systems (MASs) focus on the joint behavior of autonomous agents. In the past decades, researchers in various fields focused on how agents cooperate with each other and revealed many interesting phenomena [3, 14]. A fundamental problem in MASs is designing a control law for each agent that solely relies on neighborhood information, so that the networked system can achieve specific tasks such as formation, swarming or consensus. Several prestigious papers [4, 11] have further highlighted the important and fundamental problems the cooperative control of MASs suffers from.

Many remarkable results have been reported to deal with various multi-agent distributed control and coordination tasks, e.g., consensus/synchronization [20], formation control [8, 36], bipartite consensus [18, 39], and containment control [7, 19]. Since the agents must agree on their respective tasks in cooperative control, the consensus control of a multi-agent system (MAS) has been a popular research topic. Currently, there are mainly two consensus control strategies: the behavior-based (or leaderless) strategy [17, 24] and the leader-follower strategy [9, 43]. The main task of a consensus control problem is to design appropriate distributed consensus protocols to achieve consensus. However, designing distributed protocols is challenging due to

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the interaction between agents [16].

To date, the consensus problem has been extensively studied in the control community. For instance, in [24, 26], the consensus problems for some simple linear MASs were investigated. Since then, the literature has addressed the consensus control for the case with noises [51], for general linear homogeneous MASs [15, 34, 46], some nonlinear MASs, such as Lipschitz nonlinear systems [31], Euler-Lagrange systems [23], rigid body systems [27], nonlinear MASs with compasses[22] and fractional MASs [44]. Note that the well-known backstepping technique originally developed in [13] for nonlinear adaptive control design is still effective and quite popular for cooperative control design and analysis of MASs [40]. Furthermore, the output regulation technique is also a powerful tool for cooperative control design and analysis, and many remarkable results have been published [35, 41].

Adaptive control methods are widely used in various fields [42] in which the model reference adaptive control (MRAC) technique has attracted significant attention since it can simultaneously realize online parameter estimation and asymptotic tracking control for systems with large parametric/structural uncertainties [1, 10, 30, 37, 45, 48, 49]. Many key problems in cooperative control theory and applications have been well handled by using MRAC-based control methods [5, 6, 21, 47, 50]. Research on distributed MRAC for open-loop reference models has been done in [25]. Moreover, [30] studied the adaptive leader-follower consensus problem for MASs with general linear dynamics and switching topologies. In [5], the authors considered that the leader's external input is not shared with any follower agent and proposed a new external input estimator in a hierarchical and cooperative manner. All these results are developed under the distributed MRAC framework.

However, how to develop a fully distributed output feedback MRAC is still an open research case. Actually, after reviewing the distributed MRAC literature, we find that the existing distributed MRAC results mainly used state feedback to solve the state consensus problems under the well-known matching condition. The latter condition requires the dynamics of the followers and the leader to meet some structural matching equations from which the ideal parameters of the nominal control laws can be calculated. The matching condition with respect to most of the real control systems is quite restrictive, and largely constrains the application range of such methods. Thus, one key technical problem that must be concerned is how to relax the restrictive matching conditions, especially for the distributed MRAC. Moreover, to our knowledge, a fully distributed output feedback MRAC has never been reported yet, which faces several key technical problems to be concerned. Such problems are (i) how to estimate the unknown parameters of all followers by only using their own input and output? (ii) how to design a distributed MRAC law for each follower by only using the local input and output information? (iii) how do all leader-follower tracking errors converge to zero without persistent excitation? These technical problems have not been addressed in the literature yet. Hence, this paper systematically addresses the distributed output feedback MRAC problem and solves the above technical problems. Specifically, we develop a fully distributed output feedback MRAC scheme without requiring the restrictive matching condition. Particularly, the asymptotic convergence of the leader-follower consensus is achieved.

Overall, this work's main contributions and novelties are as follows.

(i) A linearly parameterized output feedback adaptive control framework is established to address the distributed leader-follower output consensus problem for linear MASs in general input-output forms. Each agent's dynamics have different pole/zero polynomials and different orders, with all coefficients being unknown.

(ii) A fully distributed output feedback adaptive control law is developed for the considered MASs, where the adaptive control law of each follower solely relies on the local input and output information without requiring the state observer and the restrictive structural matching condition on the followers and leader dynamics commonly used in the literature.

(iii) To establish the distributed output matching equation for each follower, some auxiliary systems are introduced to generate filtered signals of individual signals and neighbors' outputs. Such filtered signals are crucial to constructing the distributed matching equations from which the adaptive parameters used in the adaptive control laws can always be derived.

(iv) The closed-loop stability and asymptotic output consensus analysis are conducted by using a gradient-based framework independent of Lyapunov functions. Particularly, the leader-follower output tracking error and its derivatives up to the n^*-th order converge to zero asymptotically without persistent excitation, which has not yet been reported in the literature.

The remainder of this paper is organized as follows. Section 1 introduces the notation employed, and Section 2 provides the problem statement and the preliminaries. Section 3 introduces the distributed output feedback MRC design and the corresponding theoretical results for providing the basic idea. Section 4 is the main part of this paper presenting the adaptive control details where the coefficients are unknown, and Section 5 presents two simulation examples to illustrate our algorithm's performance. Finally, Section 6 concludes this paper.

Notation: In this paper, $\mathbb{R}$ denotes the sets of real numbers. Let s denote the differential operator, i.e. $s[x](t) = \dot{x}(t)$ with $x(t) \in \mathbb{R}^n$, $t \geq t_0$. With L^∞, L^2 and L^1, we denote three signal spaces defined as $L^\infty = \{x(t) : \|x(\cdot)\|_\infty < \infty\}$, $L^2 = \{x(t) : \|x(\cdot)\|_2 < \infty\}$ and $L^1 = \{x(t) : \|x(\cdot)\|_1 < \infty\}$ with $\|x(\cdot)\|_\infty = \sup_{t\geq t_0} \|x(t)\|_\infty$, $\|x(\cdot)\|_2 = \left(\int_{t_0}^{\infty} \|x(t)\|_2^2 dt\right)^{1/2}$ and $\|x(\cdot)\|_1 = \int_{t_0}^{\infty} \|x(t)\|_1 dt$, respectively.

2. Problem statement.

This section formulates the system model, the control objective, the design conditions, and the technical issues to be solved.

2.1. System model.

The MAS considered in this paper is described by the following input-output form:

$$P_i(s)[y_i](t) = k_{pi} Z_i(s)[u_i](t), \ t \geq 0, \ i = 1, \ldots, N, \tag{2.1}$$

where N is the number of the agents, $y_i(t) \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ are the output and input of the i-th follower, respectively, k_{pi} is a constant referred to as the high frequency gain, and $P_i(s)$ and $Z_i(s)$ are the pole and zero polynomials with unknown coefficients, degree n_i and m_i, respectively, i.e.,

$$\begin{aligned} P_i(s) &= s^{n_i} + p_{i,n_i-1} s^{n_i-1} + \cdots + p_{i1} s + p_{i0}, \\ Z_i(s) &= s^{m_i} + z_{i,m_i-1} s^{m_i-1} + \cdots + z_{i1} s + z_{i0}. \end{aligned}$$

It should be noted that n_i and n_j, as well as m_i and m_j, can be different for $i \neq j$, with $i, j = 1, \ldots, N$.

The leader $y_0(t)$'s dynamic model is

$$P_m(s)[y_0](t) = r(t), \tag{2.2}$$

where $P_m(s)$ is a stable polynomial of degree n^*, and $r(t)$ is a bounded and piecewise continuous reference input signal for the leader.

Actually, (2.2) can be chosen more general as: $P_m(s)[y_0](t) = Z_m(s)[r](t)$, with $Z_m(s)$ and $P_m(s)$ being two given zero and pole polynomials. But, the design and analysis for more general cases are similar to that for the case of (2.2). Therefore, for simplicity of presentation, here we choose (2.2) to conduct the distributed MRAC design and analysis. The reader can refer to [37] and [10] for further details.

Next, it is important to clarify the necessity of using the input-output form (2.1) to establish a distributed MRAC framework. Some black-box systems may not afford to build a state-space system model when no information about the internal state variables is available. However, establishing a simple input-output model without containing internal state variables is possible for such black-box systems. In this case, the input-output information is adequate for the MRAC and distributed MRAC control design and stability analysis. However, a potentially arising question is that as long as an input-output model is established, one may derive its state-space realization and still use state-space-based methods to conduct the control design and analysis. Indeed, the state-space model can be derived from the input-output model. However, from a practical viewpoint, the state-space model may sometimes be unsuitable for designing the controller because the state variables generally do not have explicit physical meanings. Therefore, addressing the cooperative control problems by using the input-output models (2.1)-(2.2) is significant.

Communication graph. Let the MAS be described by (2.1)-(2.2). The communications between these $N+1$ agents are modeled as a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{v_0, \ldots, v_N\}$ is the set of nodes with v_0 representing the leader, $v_i, i = 1, \ldots, N$, representing the i-th follower, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ being the set of edges of $\mathcal{G}$. The directed edge (v_j, v_i) represents a unidirectional communication channel from agent v_j to agent v_i, i.e., agent v_i can obtain the output information from agent v_j, but not vice versa. The neighborhood of agent v_i, $i = 0, \ldots, N$, is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. A directed sequence of the edges $(v_{i1}, v_{i2}), (v_{i2}, v_{i3}), \ldots, (v_{i,k-1}, v_{ik})$ is called a path from node v_{i1} to node v_{ik}. A directed tree is a directed graph where each node except for the root node has a single neighbor, and the root node is a source node. A spanning tree of $\mathcal{G}$ is a directed tree whose node set is $\mathcal{V}$. Its edge set is a subset of $\mathcal{E}$. Moreover, (v_i, v_i) is called a self-loop. This study assumes a simple graph, i.e., the graph has no self-loops or multiple arcs.

2.2. Control objective and design conditions.

Control objective. For the MAS (2.1)-(2.2), the control objective is to design a distributed output feedback MRAC law solely using local input and output information so that the closed-loop system is stable and of the higher-order output consensus properties:

$$\lim_{t\to\infty}(y_i(t) - y_0(t))^{(j)} = 0, i = 1, \ldots, N,\ j = 0, \ldots, n^*, \tag{2.3}$$

where $y^{(j)}(t)$ denotes the j-th derivative of $y(t)$.

Assumptions. To meet the control objective given by (2.3), we present the following assumptions:

(A1) All $Z_i(s)$, $i = 1, \ldots, N$, are stable polynomials.
(A2) The relative degree of i-th follower is $n_i - m_i = n^*$ for $i = 1, \ldots, N$.
(A3) An upper bound on n_i, denoted as $\bar{n}$, is known.
(A4) The leader input $r(t)$ satisfies $\dot{r}(t) \in L^\infty$.
(A5) The directed graph $\mathcal{G}$ has at least one spanning tree with v_0 being the parent.

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It is well known that the usual MRAC systems require the zeros of the control system to be stable, which is a consequence of zero-pole cancellations occurring in the MRAC systems. In this case, the MRAC law will cancel and replace the control system's zeros with the reference model's. For stability, such cancellations must be stable. In other words, the control system must be minimum-phase. Moreover, the control system's relative degree must equal the reference system's degree to guarantee model matching, which is necessary for tracking target even if when the system parameters are known [37]. For a distributed MRAC design, Assumptions (A1) and (A2) are regarded as extensions of the minimum-phase condition and the model-matching condition in the usual MRAC systems. Moreover, Assumption (A3) is required for constructing a parameterized system model for parameter adaptation. Besides, Assumptions (A1)-(A3) are the traditional design conditions in the usual MRAC systems, and Assumption (A4) is a relaxed design condition on the reference system, which is used to ensure higher-order output consensus. Finally, Assumption (A5) is a typical design condition for the output consensus control that is commonly used in the literature.

2.3. Comparisons and technical issues to be solved.

Comparison to cooperative output regulation. The linear cooperative output regulation problem was first formally formulated and solved using a distributed observer approach on a static network in [32] and then on a jointly connected switched network in [33]. In order to address the design condition where each follower possesses knowledge of the leader's system matrix, the literature [2] investigates the linear cooperative output regulation problem on static networks using an adaptive distributed observer approach. The output regulation based cooperative control has been systematically studied in the control community. Generally speaking, the standard output regulation based cooperative control method typically relies on the existence of a solution for the regulator equations, which fundamentally distinguishes it from the well-known MRAC technique. This is the reason why the establishment of a fully distributed output feedback MRAC framework for cooperative control remains an imperative, necessitating our attention and focus.

Comparison to distributed MRAC. As mentioned in the Introduction, distributed MRAC methods are now applied to multi-agent linear time-invariant systems. However, the existing literatures [5, 21, 30, 47, 50] mainly focus on the MASs described by the state feedback for state tracking. The followers' models are of the basic form: $\dot{x}_i = A_i x_i + B_i u_i,\ i = 1, ..., N$, where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{m_i}$, $i = 1, ..., N$, are the state vectors and input vectors of the followers, A_i and B_i, $i = 1, ..., N$, are unknown constant matrices of appropriate dimensions. The leader model is of the basic form: $\dot{x}_0 = A_0 x_0 + B_0 u_0$, where $x_0 \in \mathbb{R}^n$ is the state vector, $u_0 \in \mathbb{R}^m$ is the bounded reference input, and A_0 and B_0 are constant matrices, with A_0 being stable.

The control objective is to find a distributed MRAC law that ensures closed-loop stability and asymptotic state consensus $\lim_{t\to\infty}(x_i - x_0) = 0$. To achieve the control objective, an essential condition, known as the structural matching condition, is as follows. (i) For each follower v_i, there exists a constant matrix K^*_{1ij} and a nonsingular constant matrix K^*_{4i} of appropriate dimensions such that

$$A_{ei} = A_i + B_i K^{*T}_{1ij},\ B_{ei} = B_i K^*_{4i}, \tag{2.4}$$

where A_{ei} is a stable and known matrix, and B_{ei} is a known matrix for $i = 1, ..., N$. (ii) For each pair of $(v_i, v_j) \in \mathcal{E}$, there exists a constant matrix K^*_{2ij} and K^*_{3ij} of appropriate dimensions such that for $i = 1, ..., N$,

$$A_j = A_i + B_i K_{3ij}^{*T},\ B_j = B_i K_{2ij}^{*}. \tag{2.5}$$

The readers can refer to [30] for further details on the matching condition (2.4)-(2.5). Note that state consensus is a strong control objective. When state consensus is achieved, the followers can track the arbitrary behaviors of the leader, which requires structural similarities among all agents. Such structural similarities are modeled as the matching condition (2.4)-(2.5). However, the latter condition is restrictive for many applications, and largely restricts the application range of the consensus methods.

Technical issues to be solved. Considering that it is sufficient to achieve output consensus for most applications, this paper focuses on addressing how to develop a fully distributed output feedback MRAC scheme to ensure asymptotic output consensus for the MAS (2.1)-(2.2) without requiring the restrictive matching conditions just like (2.4)-(2.5). The basic idea of MRAC is to design an adaptive control law that ensures the closed-loop system matches any given reference system. Inspired by this, for the distributed output feedback MRAC, the agents that are connected to the leader follow the reference system (i.e., the leader model). However, the agents that are not connected to the leader do not have an available reference system. Thus, the first technical problem is designing virtual reference systems for the agents, especially for those not connected to the leader. Then, a potentially arising question is how to guarantee that the agents with virtual reference systems can achieve leader-follower output consensus. Moreover, the third technical problem is accomplishing the higher-order tracking properties (2.3). In a word, to establish a fully distributed output feedback MRAC framework, the following technical problems must be solved:

(i) How to design the virtual reference models for all followers and construct the plant-model matching equations, especially those that are not connected to the leader, by solely using the local input and output information?

(ii) Given that the agents could follow the virtual reference systems asymptotically, how to eventually realize leader-follower output consensus for the whole MAS (2.1)-(2.2)? Especially, asymptotic output consensus is required, which leads to more difficulties for adaptive control design and analysis.

(iii) The current results of the distributed leader-follower control indicate that the asymptotic state/output consensus property can be ensured. However, under the usual design conditions, how to ensure some higher-order output consensus as shown in (2.3)? To our knowledge, this problem has never been addressed in the literature.

3. Distributed output feedback MRC design. This section provides the basic idea of the distributed output feedback MRAC framework through a distributed model reference control (MRC) design, assuming all system parameters are known. The design contains four steps: (i) deriving the distributed MRC law structure, (ii) constructing virtual reference inputs, (iii) calculating the control law parameters, and (iv) conducting system performance analysis.

Step 1: Distributed MRC law structure. Given that all system parameters are known, we design the distributed MRC law for the i-th agent, $i = 1, \ldots, N$, as

$$u_i(t) = \theta_{1i}^{*T}\omega_{1i}(t) + \theta_{2i}^{*T}\omega_{2i}(t) + \theta_{3i}^{*}\omega_{3i}(t) + \theta_{20i}^{*}y_i(t), \tag{3.1}$$

where $\theta_{1i}^* \in \mathbb{R}^{\bar{n}-1}, \theta_{2i}^* \in \mathbb{R}^{\bar{n}-1}, \theta_{3i}^* \in \mathbb{R}$ and $\theta_{20i}^* \in \mathbb{R}$ are constant parameters to be specified, and

$$\omega_{1i}(t) = \frac{a(s)}{\Lambda_{ci}(s)}[u_i](t) \in \mathbb{R}^{\bar{n}-1},\ \omega_{2i}(t) = \frac{a(s)}{\Lambda_{ci}(s)}[y_i](t) \in \mathbb{R}^{\bar{n}-1}, \tag{3.2}$$

with $a(s) = \left[1, s, \ldots, s^{\bar{n}-2}\right]^T \in \mathbb{R}^{\bar{n}-1}$ and $\Lambda_{ci}(s) = s^{\bar{n}-1} + \lambda_{i,\bar{n}-2}^c s^{\bar{n}-2} + \cdots + \lambda_{i1}^c s + \lambda_{i0}^c$ representing an arbitrary monic Hurwitz polynomial. The signals $\omega_{1i}(t)$ and $\omega_{2i}(t)$ are obtained through filtering $u_i(t)$ and $y_i(t)$ by the stable filter $\frac{a(s)}{\Lambda_{ci}(s)}$, respectively.

Remark 3.1. Since $\Lambda_{ci}(s)$ in (3.2) is monic and of degree $\bar{n}-1$ and the maximum degree of the vector $a(s)$ is $\bar{n}-2$, each element of the vector $\frac{a(s)}{\Lambda_{ci}(s)}$ is strictly proper, i.e., the degree of the numerator $a(s)$ is strictly less than that of the denominator $\Lambda_{ci}(s)$. Thus, there does not exist any algebraic loop in the control law (3.1).

In traditional MRAC, $\omega_{3i}(t)$ corresponds to the reference system input. Since each agent receives signals from its neighbors, and the number of neighbors N_i is known, we design $\omega_{3i}(t)$ as:

$$\omega_{3i}(t) = \begin{cases} \dfrac{1}{N_i} \displaystyle\sum_{v_j \in \mathcal{N}_i} r_j(t), & v_0 \notin \mathcal{N}_i, \\ r(t), & v_0 \in \mathcal{N}_i, \end{cases} \tag{3.3}$$

where $r_j(t)$, $j = 1, ..., N_i$, are auxiliary signals to be designed.

From (3.3), for agents connected to the leader, the leader's input $r(t)$ is directly used as $\omega_{3i}(t)$, enabling them to follow the leader as in traditional MRAC. For agents not connected to the leader, $r(t)$ is unavailable. To solve this, we design the auxiliary signal $\frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i} r_j(t)$ as $\omega_{3i}(t)$, which acts as a virtual reference. Designing this virtual reference and ensuring all agents can follow the leader are key challenges addressed in this paper. Next, we explain how to obtain $r_j(t)$ to construct $\omega_{3i}(t)$.

Step 2: Virtual reference input construction. As mentioned in Appendix A, traditional model reference control requires an additional reference signal $r(t) = P_m(s)[y_m](t)$, which is the sum of some derivative information of the tracked signal. Inspired by this, if the derivatives $y_j^{(k)}(t)$, $k=1,..,n^*$ with respect to the j-th agent are known, we design $r_j(t)$ as

$$r_j(t) = \Psi(s)[y_j](t) \tag{3.4}$$

with $\Psi(s) = s^{n^*} + \psi_{n^*-1}s^{n^*-1} + \cdots + \psi_1 s + \psi_0$ being some chosen monic Hurwitz polynomials of degree n^*. However, $y_j^{(k)}(t)$ is generally difficult to be obtained. Hence, using (3.4) to obtain $r_j(t)$ is inappropriate. Thus, we present a construction method to obtain $r_j(t)$ using only u_j and y_j. For simplicity, we change the subscript from j to i, and define two vectors:

$$\begin{aligned} \theta_{pi}^* &= [k_{pi}z_{i0}, k_{pi}z_{i1}, \ldots, k_{pi}z_{i,m_i-1}, k_{pi}, -p_{i0}, -p_{i1}, \\ &\quad \ldots, -p_{i,n_i-2}, -p_{i,n_i-1}]^T \in \mathbb{R}^{n_i+m_i+1}, \end{aligned} \tag{3.5}$$

$$\begin{aligned} \phi_i(t) &= \left[\frac{1}{\Lambda_{ei}(s)}[u_i](t), \frac{s}{\Lambda_{ei}(s)}[u_i](t), \ldots, \frac{s^{m_i-1}}{\Lambda_{ei}(s)}[u_i](t), \right. \\ &\quad \frac{s^{m_i}}{\Lambda_{ei}(s)}[u_i](t), \frac{1}{\Lambda_{ei}(s)}[y_i](t), \frac{s}{\Lambda_{ei}(s)}[y_i](t), \\ &\quad \left. \ldots, \frac{s^{n_i-2}}{\Lambda_{ei}(s)}[y_i](t), \frac{s^{n_i-1}}{\Lambda_{ei}(s)}[y_i](t)\right]^T \in \mathbb{R}^{n_i+m_i-1}, \end{aligned} \tag{3.6}$$

where $\Lambda_{ei}(s) = s^{n_i} + \lambda^e_{i,n_i-1}s^{n_i-1} + \cdots + \lambda^e_{i1}s + \lambda^e_{i0}$ representing an arbitrary monic Hurwitz polynomial. Then, ignoring the exponentially decaying signal, the system (2.1) can be expressed as

$$y_i(t) - \frac{\Lambda_{i,n_i-1}(s)}{\Lambda_{ei}(s)}[y_i](t) = \theta^{*T}_{pi}\phi_i(t) \tag{3.7}$$

with $\Lambda_{i,n_i-1}(s) = \lambda^e_{i,n_i-1}s^{n_i-1} + \cdots + \lambda^e_{i1}s + \lambda^e_{i0}$. To design $r_j(t)$, we first give the following lemma demonstrating a key property of $y_i^{(j)}(t), i = 1,\ldots,N, j = 1,\ldots,n^*$.

Lemma 3.2. *For $y_i^{(j)}(t)$, $j = 1,\ldots,n^*$, it can be expressed by $y_i^{(k)}(t), k=0,\ldots,j-1$, $\frac{s^k}{\Lambda_{ei}(s)}[u_i](t), k = 1+m_i,\ldots,j+m_i, \theta^*_{pi}, \phi_i(t)$, and $y_i(t)$.*

Proof. The proof is given in Appendix B. □

Based on Lemma 3.2, we recursively obtain that $y_i^{(j)}(t)$, $j = 1,\ldots,n^*$, can be expressed by $\frac{s^k}{\Lambda_{ei}(s)}[u_i](t)$ for $k = 1+m_i,\ldots,j+m_i$, $\theta^*_{pi}, \phi_i(t)$, and $y_i(t)$. Thus, we express $y_i^{(j)}(t), j = 1,2,\ldots,n^*$, as

$$y_i{}^{(j)} = H_{ij}\left(y_i, \frac{s^{1+m_i}}{\Lambda_{ei}(s)}[u_i],\ldots,\frac{s^{j+m_i}}{\Lambda_{ei}(s)}[u_i], \theta^*_{pi}, \phi_i\right). \tag{3.8}$$

As demonstrated in the proof of Lemma 3.2, H_{ij} is obtained by applying a filter related to $\Lambda_{ei}(s)$ to the original input-output system. Its form depends solely on $\Lambda_{ei}(s)$. If $\Lambda_{ei}(s)$ is predetermined, then H_{ij} is a known mapping. Consequently, $H_{ij}, i = 1,\ldots,N, j = 1,\ldots,n^*$, are known and smooth mappings with respect to its variables. It should be noted that from (3.4), we derive an analytical expression for $r_i(t)$ as

$$r_i = \sum_{j=0}^{n^*}\psi_j H_{ij}\left(y_i, \frac{s^{1+m_i}}{\Lambda_{ei}(s)}[u_i],\ldots,\frac{s^{j+m_i}}{\Lambda_{ei}(s)}[u_i], \theta^*_{pi}, \phi_i\right), \tag{3.9}$$

where ψ_k, $k = 1,...,n^*$, are constant parameters with $\psi_{n^*} = 1$ such that $s^{n^*} + \psi_{n^*-1}s^{n^*-1} + \cdots + \psi_1 s + \psi_0$ is a Hurwitz polynomial.

Remark 3.3. From (3.9), we see that $r_i(t)$ depends on the unknown vector $\theta^*_{p_i}$. For the adaptive control case, we construct an estimate of $r_i(t)$ that will no longer depend on any unknown information (see Section 4). Besides, to estimate the higher-order derivatives of $y_i(t)$, one may employ a standard high-gain differential observer [12]. Even though the high-gain observer design is simple and easy to implement, using this observer is difficult to realize asymptotic output consensus, and involves the high-gain issue. We propose a linear parametrization-based estimation method based on this consideration to derive the $r_i(t)$'s estimate and achieve the asymptotic output consensus. Finally, it is worth noting that by (3.1), (3.3) and (3.9), it is known that each agent's controller makes use of only its own and its neighbors' information and does not need the global information of the leader.

From (3.1), it is evident that the nominal control law for each follower solely relies on local input and output information, and does not depend on global leader information.

Step 3: Calculation of θ^*_{1i}, θ^*_{2i}, θ^*_{3i}, and θ^*_{20i}. Now, we construct some plant-model output matching equations from which θ^*_{1i}, θ^*_{2i}, θ^*_{3i}, and θ^*_{20i} can be calculated.

Motivated by the usual output feedback MRC in [37], we derive the distributed version of the plant-model output matching equations as follows:

LEMMA 3.4. *For the i-th agent connected to the leader, there exist constants* $\theta_{1i}^*, \theta_{2i}^*, \theta_{20i}^*, \theta_{3i}^*$ *such that*

$$\begin{aligned}(3.10)\qquad &\theta_{1i}^{*T} a(s)P_i(s) + \left(\theta_{2i}^{*T} a(s) + \theta_{20i}^*\Lambda_{ci}(s)\right) k_{pi} Z_i(s)\\ =&\Lambda_{ci}(s)\left(P_i(s) - k_{pi}\theta_{3i}^* Z_i(s) P_m(s)\right);\end{aligned}$$

and for the i-th agent not connected to the leader, there exist constants $\theta_{1i}^*, \theta_{2i}^*, \theta_{20i}^*, \theta_{3i}^*$ *such that*

$$\begin{aligned}(3.11)\qquad &\theta_{1i}^{*T} a(s)P_i(s) + \left(\theta_{2i}^{*T} a(s) + \theta_{20i}^*\Lambda_{ci}(s)\right) k_{pi} Z_i(s)\\ =&\Lambda_{ci}(s)\left(P_i(s) - k_{pi}\theta_{3i}^* Z_i(s) \Psi(s)\right)\end{aligned}$$

where $a(s)$ *and* $\Psi(s)$ *are defined below (3.2) and (3.4), respectively.*

Proof. The proof is similar to that of Lemma A.2 in Appendix A, and thus, omitted here. For details, one may refer to [37]. ☐

Remark 3.5. These matching equations always have non-trivial analytical solutions, and one can choose the solution $\{\theta_{1i}^*, \theta_{2i}^*, \theta_{20i}^*, \theta_{3i}^*\}$ to (3.10)-(3.11) from

$$(3.12)\qquad \theta_{1i}^{*T} a(s) = \Lambda_{ci}(s) - Q(s)Z_i(s),\ \theta_{2i}^{*T} a(s) + \theta_{20i}^*\Lambda_{ci}(s) = -\theta_{3i}^* R_i(s),$$

and $\theta_{3i}^* = \frac{1}{k_{pi}}$, where $Q(s)$ is the quotient of $\frac{\Lambda_{ci}(s)P_m(s)}{P_i(s)}$ and $R_i(s) = \Lambda_{ci}(s)P_m(s) - Q(s)P_i(s)$ for (3.10), and $Q(s)$ is the quotient of $\frac{\Lambda_{ci}(s)\Psi(s)}{P_i(s)}$ and $R_i(s) = \Lambda_{ci}(s)\Psi(s) - Q(s)P_i(s)$ for (3.11).

The parameters $\theta_{1i}^*, \theta_{2i}^*, \theta_{20i}^*, \theta_{3i}^*$ in Lemma 3.4 can be called distributed matching parameters, as with these parameters, the distributed MRC law (3.1) matches all followers to the leader, as shown subsequently.

Step 4: System performance analysis. To proceed, we first define the local output tracking error as

$$(3.13)\qquad e_i(t) = y_i(t) - \frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i} y_j(t),\quad i = 1,\ldots,N,$$

where N_i is the number of the neighbors of agent v_i. Such a local output tracking error measures the disagreement between the follower i and the average of its neighbors on the output because it is essential to characterize the consensus level of the follower and the leader. The motivation of defining such a local state tracking error is shown as follows:

LEMMA 3.6. *Under Assumption (A5), if* $e_i(t)$ *is bounded, then* $y_i(t)$ *is bounded for all* $i = 1,\ldots,N$. *Further if for any* $j = 1,\ldots,n^*$, $\lim_{t\to\infty} e_i^{(j)}(t) = 0$ *holds (or exponentially) for all* $i = 1,...,N$, *then* $\lim_{t\to\infty}(y_i(t) - y_0(t))^{(j)} = 0$ *holds (or exponentially) for all* $i = 1,\ldots,N$.

Proof. Performing a proof similar to that for Lemma 4.1 in [29], one can verify this lemma. ☐

From Lemma 3.6, global higher-order leader-follower consensus properties can be achieved as long as the higher-order derivatives of all local tracking errors (3.13) converge to zero as time tends to infinity. According to this lemma, the following theorem clarifies the closed-loop stability and output consensus performance.

THEOREM 3.7. *Under Assumptions (A1), (A2) and (A5), the distributed MRC law (3.1) configured with* $\theta_{1i}^*, \theta_{2i}^*, \theta_{20i}^*, \theta_{3i}^*$ *in Lemma 3.4 ensures that all closed-loop signals are bounded and the tracking errors* $y_i(t) - y_0(t)$, $i = 1,...,N$, *and their derivatives up to the* n^**-th order converge to zero exponentially as* $t \to \infty$.

Proof. For all agents $v_i \in \{v_i : v_0 \in \mathcal{N}_i\}$, the leader v_0 can be regarded as the reference output. Thus, based on Theorem A.3 in Appendix A , one can verify that the input $u_i(t) = \theta_{1i}^{*T}\omega_{1i}(t) + \theta_{2i}^{*T}\omega_{2i}(t) + \theta_{20i}^* y_i(t) + \theta_{3i}^* r(t)$ ensures that the signals of the agent v_i are bounded, and $y_i(t) - y_0(t)$, $i = 1,...,N$, and their derivatives up to the n^*-th order converge to zero exponentially.

For the agent $v_i \notin \{v_i : v_0 \in \mathcal{N}_i\}$, by Lemma 3.4, we first prove that $e_i(t)$ converges to zero exponentially. Operating both sides of (3.11) on $y_i(t)$, we have

$$\begin{aligned} &\theta_{1i}^T a(s)P_i(s)[y_i](t) + \left(\theta_{2i}^T a(s) + \theta_{20i}\Lambda_{ci}(s)\right) k_{pi} \\ &Z_i(s)[y_i](t) = \Lambda_{ci}(s)(P_i(s) - k_{pi}\theta_{3i}Z_i(s)\Psi(s))[y_i](t). \end{aligned} \tag{3.14}$$

Moreover, with some manipulations on (3.1), we have

$$\begin{aligned} \Lambda_{ci}(s)[u_i](t) \;=\; &\theta_{1i}^T a(s)[u_i](t) + \theta_{2i}^T a(s)[y_i](t) + \theta_{3i}\Lambda_{ci}(s)\Psi(s)[\frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i} y_j](t) \\ &+\Lambda_{ci}(s)\theta_{20i}[y_i](t) + \Lambda_{ci}(s)\left[\epsilon_{\Lambda_{ci}}\right](t), \end{aligned} \tag{3.15}$$

where $\epsilon_{\Lambda_{ci}}(t)$ is an exponentially decaying signal associated with the initial conditions. Then, we have

$$\begin{aligned} &k_{pi}Z_i(s)\Lambda_{ci}(s)\left[u_i\right](t) = P_i(s)\Lambda_{ci}(s)\left[y_i\right](t) \\ &= k_{pi}Z_i(s)\Lambda_{ci}(s)\theta_{20i}\left[y_i\right](t) + k_{pi}Z_i(s)\Lambda_{ci}(s)\left[\epsilon_{\Lambda_{ci}}\right](t) \\ &+k_{pi}Z_i(s)\theta_{3i}\Lambda_{ci}(s)\Psi_i(s)[\frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i} y_j](t) \\ &+k_{pi}Z_i(s)\left(\theta_{1i}^T a(s)\left[u_i\right](t) + \theta_{2i}^T a(s)\left[y_i\right](t)\right). \end{aligned} \tag{3.16}$$

Combining (3.16) and (3.14), together with $P_i(s)[y_i](t) = k_{pi}Z_i(s)[u_i](t)$, indicates that

$$\Lambda_{ci}(s)\Psi(s)Z_i(s)[y_i - \frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i} y_j](t) = -k_{pi}Z_i(s)\Lambda_{ci}(s)\left[\epsilon_{\Lambda_{ci}}\right](t). \tag{3.17}$$

Since $\Lambda_{ci}(s), \Psi(s)$ and $Z_i(s)$ are all stable polynomials and the degree of $\Psi(s)$ is n^*, we conclude that for $l = 0, 1, \ldots, n^*$,

$$(y_i(t) - \frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i} y_j(t))^{(l)} \to 0, \text{ exponentially.} \tag{3.18}$$

According to Lemma 3.6, (3.18) suggests that the higher order exponential leader-follower consensus (2.3) is achieved. This also implies that $y_i(t) \in L^\infty$ due to the boundedness of $y_0(t)$.

Now, we prove $u_i(t)$, $i = 1, \ldots, N$, are also bounded. Using (2.1) and (3.17), we have $k_{pi}Z_i(s)^2\Lambda_{ci}(s)\Psi(s)[u_i(t)] = P_i(s)\Lambda_{ci}(s)Z_i(s)[\frac{1}{n_i}\sum_{v_j\in\mathcal{N}_i} r_j](t) + \epsilon_{1i}(t)$ with $\epsilon_{1i}(t) = -k_{pi}Z_i(s)\Lambda_{ci}(s)\left[\epsilon_{\Lambda_{ci}}\right](t)$. Since $\Lambda_{ci}(s)$, $\Psi(s)$ and $Z_i(s)$ are all stable, we can derive

$$u_i(t) = \frac{P_i(s)}{k_{pi}Z_i(s)\Psi_i(s)}\left[\frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i} r_j\right](t) + \epsilon_{2i}(t),$$

where $\epsilon_{2i}(t)$ is an exponentially decaying signal associated with initial conditions. Note that $\frac{P_i(s)}{k_{pi}Z_i(s)\Psi(s)}$ is stable and proper, i.e. the degree of the numerator $P_i(s)$ is not greater than that of the denominator $k_{pi}Z_i(s)\Psi(s)$. Thus, if $\sum_{v_i\in\mathcal{N}_i} r_j \in L^\infty$, then $u_i(t) \in L^\infty$.

Let l_i denote the length of the longest directed path for the leader v_0 to the node v_i. Suppose that there exists a follower v_k such that r_k is unbounded. Then, there exists a neighbor v_{k_j} of v_k such that r_{k_j} is unbounded and $l_{k_j} < l_k$. From Assumption (A5), and by repeating this analysis for up to l_k steps, we conclude that the reference signal of the leader $r(t)$ is unbounded, which is a contradiction. Therefore, $r_i(t) \in L^\infty,\ i = 1, \ldots, N$, and so are the control $u_i(t)$. This completes the proof. □

Remark 3.8. Equation (3.17) shows that the convergence rate is influenced by the roots of a certain polynomial, with larger roots leading to faster convergence speed. However, large roots can cause initial output fluctuations. Therefore, the choice of Λ_{ei} and Λ_{ci} should consider both the convergence speed and the transient performance of the system.

So far, we have provided a basic **distributed MRC framework** for the MAS (2.1)-(2.2) which is fundamental for the **distributed MRAC design** addressed next.

4. Distributed output feedback MRAC design. This section develops a distributed output feedback indirect MRAC scheme for the MAS (2.1)-(2.2), where the parameters p_{ij}, z_{ij}, and k_{pi} are unknown. Specifically, we construct the distributed output feedback MRAC law, with the distributed indirect MRAC design procedure comprising five steps: (i) distributed MARC law construction, (ii) plant parameter estimation, (iii) controller parameter calculation, (iv) virtual reference input signal estimation, and (v) stability performance analysis.

Step 1: Distributed MARC law structure. The distributed MRAC law is designed as

$$u_i(t) = \theta_{1i}^T(t)\omega_{1i}(t) + \theta_{2i}^T(t)\omega_{2i}(t) + \theta_{3i}(t)\hat{\omega}_{3i}(t) + \theta_{20i}(t)y_i(t), \tag{4.1}$$

where $\theta_{1i}(t)$ and $\theta_{2i}(t)$ are estimates of θ_{1i}^* and θ_{2i}^* in Lemma 3.4, respectively, $\theta_{3i}(t)$ is an estimate of $\frac{1}{k_{pi}}$, $\omega_{1i}(t)$ and $\omega_{2i}(t)$ are defined in (3.2), and $\hat{\omega}_{3i}(t)$ is an estimate of $\omega_{3i}(t)$ in (3.3).

Step 2: Plant parameter estimation. Consider the i-th follower in (2.1). The signal $\phi_i(t)$ in (3.6) can be obtained through filtering $u_i(t)$ and $y_i(t)$ by the stable filter $\frac{a_i(s)}{\Lambda_{ei}(s)}$ with $a_i(s) = \left[1, s, \ldots, s^{n_i-2}\right]^T$ and $\Lambda_{ei}(s)$ below (3.6). Similarly, $\frac{\Lambda_{i,n_i-1}(s)}{\Lambda_{ei}(s)}[y_i](t)$ in (3.7) can be obtained through filtering $y_i(t)$ by the stable filter $\frac{\Lambda_{i,n_i-1}(s)}{\Lambda_{ei}(s)}$.

Let $\theta_{pi}(t)$ be an estimate of θ_{pi}^* and define the estimation error as

$$\epsilon_i(t) = \theta_{pi}^T(t)\phi_i(t) - y_i(t) + \frac{\Lambda_{i,n_i-1}(s)}{\Lambda_{ei}(s)}[y_i](t), t \geq t_0. \tag{4.2}$$

To update $\theta_{pi}(t)$, we use the following gradient algorithm:

$$\dot{\theta}_{pi}(t) = -\frac{\Gamma_i\phi_i(t)\epsilon_i(t)}{m_i^2(t)}, \theta_{pi}(t_0) = \theta_{0i}, t \geq t_0, \tag{4.3}$$

where $\Gamma_i = \text{diag}\{\Gamma_{1i}, \gamma_{m_i+1}, \Gamma_{2i}\}$ with $\Gamma_{1i} \in \mathbb{R}^{m_i \times m_i}$, $\Gamma_{1i} = \Gamma_{1i}^T > 0, \gamma_{m_i+1} > 0$ and $\Gamma_{2i} \in \mathbb{R}^{n_i \times n_i}, \Gamma_{2i} = \Gamma_{2i}^T > 0$, θ_{0i} is an initial estimate of $\theta_{pi}^* \in \mathbb{R}^{n_i+m_i+1}$, and

$$m_i(t) = \sqrt{1 + \kappa \phi_i^T(t)\phi_i(t)},\ \kappa > 0. \tag{4.4}$$

From (3.5), we denote $\theta_{pi}(t)$ as

$$\theta_{pi}(t) = \left[\widehat{k_{pi}z_{i0}}(t), \ldots, \widehat{k_{pi}z}_{im,_i-1}(t), \hat{k}_{pi}(t), -\hat{p}_{i0}(t), \ldots, -\hat{p}_{i,n_i-1}(t)\right]^T.$$

Thus, we construct the estimates of $P_i(s)$ and $Z_i(s)$ for the i-th follower as

$$\begin{aligned} \hat{P}_i(s, \hat{p}_i) &= s^{n_i} + \hat{p}_{i,n_i-1}s^{n_i-1} + \cdots + \hat{p}_{i1}s + \hat{p}_{i0}, \\ \hat{Z}_i(s, \hat{z}_i) &= s^{m_i} + \hat{z}_{i,m_i-1}s^{m_i-1} + \cdots + \hat{z}_{i1}s + \hat{z}_{i0}, \end{aligned} \tag{4.5}$$

where $\hat{z}_i = [\hat{z}_{i0}, \ldots, \hat{z}_{i,m_i-1}]^T$ with $\hat{z}_{ij} = \frac{\widehat{k_{pi}z_{ij}}(t)}{\hat{k}_{pi}(t)}$ and $\hat{p}_i = [\hat{p}_{i0}, \ldots, \hat{p}_{i,n_i-1}]^T$ are the estimates of $z_i^* = [z_{i0}, \ldots, z_{i,m_i-1})]^T$ and $p_i^* = [p_{i0}, \ldots, p_{i,n_i-1}]^T$, respectively. To ensure $\hat{k}_{pi}(t) \neq 0$ during parameter adaptation, the parameter update law (4.3) needs to be modified by introducing some robust term, such as parameter projection, dead-zone modification, σ-modification, and so on. We omit the details due to the paper length constraints.

For the parameter $\theta_{pi}(t)$, the following lemma clarifies some properties crucial for stability analysis.

LEMMA 4.1. *The adaptive algorithm (4.3) guarantees (i) $\theta_{pi}(t), \dot{\theta}_{pi}(t), \frac{\epsilon_i(t)}{m_i(t)}$ are bounded and (ii) $\frac{\epsilon_i(t)}{m_i(t)}$ and $\dot{\theta}_{pi}(t)$ belong to L^2.*

Proof. The proof is similar to Lemma 3.1 in [37], and so, it is omitted here. □

Note that the regressor vector $\phi_i(t)$ is not required to be persistently exciting, and thus, we cannot ensure that the estimation errors $\epsilon_i(t)$ converge to zero. Nevertheless, this paper shows that the proposed distributed MRAC law (4.1) still ensures closed-loop stability and the tracking properties shown in (2.3).

Step 3: Controller parameter calculation. For the i-th agent connected to the leader, the controller parameters $\{\theta_{1i}(t), \theta_{2i}(t), \theta_{20i}(t), \theta_{3i}(t)\}$ are obtained from

$$\begin{aligned} &\theta_{1i}^T a(s)\hat{P}_i(s, \hat{p}_i) + \left(\theta_{2i}^T a(s) + \theta_{20i}\Lambda_{ci}(s)\right) k_{pi}\hat{Z}_i(s, \hat{z}_i) \\ &= \Lambda_{ci}(s)\left(\hat{P}_i(s, \hat{p}_i) - \hat{k}_{pi}\theta_{3i}\hat{Z}_i(s, \hat{z}_i)P_m(s)\right), \end{aligned} \tag{4.6}$$

and for the i-th agent not connected to the leader, the controller parameters are obtained from

$$\begin{aligned} &\theta_{1i}^T a(s)\hat{P}_i(s, \hat{p}_i) + \left(\theta_{2i}^T a(s) + \theta_{20i}\Lambda_{ci}(s)\right) k_{pi}\hat{Z}_i(s, \hat{z}_i) \\ &= \Lambda_{ci}(s)\left(\hat{P}_i(s, \hat{p}_i) - \hat{k}_{pi}\theta_{3i}\hat{Z}_i(s, \hat{z}_i)\Psi(s)\right). \end{aligned} \tag{4.7}$$

Regarding how to specifically derive $\theta_{1i}(t)$, $\theta_{2i}(t)$, $\theta_{20i}(t)$, $\theta_{3i}(t)$, the reader can refer to (3.12).

Step 4: Virtual reference input signal estimation. The signal $\hat{\omega}_{3i}(t)$ in (4.1) is designed by

$$\hat{\omega}_{3i}(t) = \begin{cases} \dfrac{1}{N_i}\displaystyle\sum_{v_j \in \mathcal{N}_i} \hat{r}_j(t), & v_0 \notin \mathcal{N}_i, \\ r(t), & v_0 \in \mathcal{N}_i, \end{cases} \tag{4.8}$$

where $\hat{r}_j(t)$ is an estimate of the signal $r_j(t)$. For simplicity, we change the subscript of $\hat{r}_j(t)$ from j to i, and design $\hat{r}_i(t)$ as

$$\hat{r}_i = \sum_{j=0}^{n^*} \psi_j H_{ij}\left(y_i, \frac{s^{1+m_i}}{\Lambda_{ei}(s)}[u_i], \ldots, \frac{s^{j+m_i}}{\Lambda_{ei}(s)}[u_i], \theta_{pi}, \phi_i\right). \tag{4.9}$$

Now, we derive the following lemma to demonstrate a convergent property of the error $\hat{r}_i(t) - r_i(t)$ under some particular conditions.

LEMMA 4.2. *For the gradient algorithm (4.3), if* $m_i(t) \in L^\infty$, $\dot{u}_i(t) \in L^\infty$ *and* $\dot{y}_i(t) \in L^\infty$, *then we have* $\dot{\hat{r}}_i(t) \in L^\infty$ *and*

$$\lim_{t\to\infty}(\hat{r}_i(t) - r_i(t)) = 0. \tag{4.10}$$

Proof. The proof of this lemma is long. Thus, we present it in Appendix B to avoid disrupting the reading flow. □

Step 5: System performance analysis. Based on the above derivations, we provide the main result of this paper, which demonstrates that the closed-loop stability and asymptotic higher-order output consensus are achieved by using the distributed MRAC law (4.1).

THEOREM 4.3. *Under Assumptions (A1)-(A5), the distributed output feedback MRAC law (4.1) ensures that all signals in the adaptive control system comprising (2.1), (2.2), (4.1) and (4.3) are bounded, and for* $i = 1, \ldots, N$,

$$\lim_{t\to\infty}(y_i(t) - y_0(t))^{(k)} = 0,\ k = 0, \ldots, n^*. \tag{4.11}$$

Proof. First, we prove that the agents connected to the leader can track the leader and generate a virtual signal $\hat{r}(t)$ satisfying $\lim_{t\to\infty}(\hat{r}(t) - r(t)) \to 0$ and $\dot{\hat{r}}(t) \in L^\infty$. For the i-th agent connected to the leader, the control law becomes $u_i(t) = \theta_{1i}^T(t)\omega_{1i}(t) + \theta_{2i}^T(t)\omega_{2i}(t) + \theta_{3i}(t)r(t) + \theta_{20i}(t)y_i(t)$. Hence, from Theorem A.4 in Appendix A, we have the closed-loop stability and $\lim_{t\to\infty}(y_i(t) - y_0(t)) = 0$. Under Assumption (A4), we have $\dot{u}_i(t) \in L^\infty$ and $\dot{y}_i(t) \in L^\infty$. Following Lemma 4.2, and combined with the closed loop stability yields $\lim_{t\to\infty}(\hat{r}_i(t) - r(t)) = 0$ and $\dot{\hat{r}}_i(t) \in L^\infty$.

Second, we prove that for the i-th agent, if the conditions $\lim_{t\to\infty}(\hat{r}_j(t) - r_j(t)) = 0$ and $\dot{\hat{r}}_j(t) \in L^\infty$ are satisfied for any $v_j \in \mathcal{N}_i$, then the following properties hold

$$\lim_{t\to\infty}(y_i(t) - \frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i} y_j(t))^{(k)} = 0, \tag{4.12}$$

for any $k = 0, \ldots, n^*, i = 1, \ldots, N$ and $\dot{\hat{r}}_i(t) \in L^\infty$. In view of the control (4.1), for any $v_j \in \mathcal{N}_i$, define

$$\hat{y}_j(t) = \frac{1}{\Psi(s)}[\hat{r}_j](t). \tag{4.13}$$

Then, ignoring the exponentially decaying signal, it follows from (4.13) that $\hat{r}_j(t) = \Psi(s)[\hat{y}_j](t)$. Substituting it into (4.8) yields $\hat{\omega}_{3i}(t) = \Psi(s)[\frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i}\hat{y}_j](t)$. Based on Theorem A.4 in Appendix A with $u_i(t) = \theta_{1i}^T(t)\omega_{1i}(t) + \theta_{2i}^T(t)\omega_{2i}(t) + \theta_{3i}(t)\hat{\omega}_{3i}(t) +$

$\theta_{20i}(t)y_i(t)$, all signals with respect to the i-th agent system are bounded and $\lim_{t\to\infty}\left(y_i(t) - \frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i}\hat{y}_j(t)\right) = 0$. Moreover, we further verify that

$$\lim_{t\to\infty}\left(y_i(t) - \frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i}\hat{y}_j\right)^{(k)} = 0, k = 0,\ldots,n^*. \tag{4.14}$$

Proving (4.14) is quite similar to that of Theorem 3.1 in [38], and thus, omitted here. Since

$$\begin{aligned}&\lim_{t\to\infty}(y_i(t) - \frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i}y_j(t))^{(k)}\\ &= \lim_{t\to\infty}(y_i(t) - \frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i}\hat{y}_j(t))^{(k)} + \lim_{t\to\infty}(\frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i}(\frac{1}{\Psi(s)}[\hat{r}_j - r_j](t))^{(k)}),\end{aligned} \tag{4.15}$$

it is sufficient to prove that for any $v_j \in \mathcal{N}_i$, the following equation holds:

$$\lim_{t\to\infty}(\frac{1}{\Psi(s)}\left[\hat{r}_j - r_j\right](t))^{(k)} = 0. \tag{4.16}$$

Let $\varepsilon_j(t) = \hat{r}_j(t) - r_j(t)$ and the k-th order time derivative of $\frac{1}{\Psi(s)}[\varepsilon_j](t)$ is $\frac{s^k}{\Psi(s)}[\varepsilon_j](t)$. Thus, with $\frac{s^k}{\Psi(s)}$ being stable and proper, if $\lim_{t\to\infty}(\hat{r}_j(t) - r_j(t)) = 0$ for $v_j \in \mathcal{N}_i$, the property (4.16) holds. Moreover, if $\dot{\hat{r}}_j(t) \in L^\infty$ for $v_j \in \mathcal{N}_i$, then $\dot{u}_i(t) \in L^\infty$ and $\dot{y}_i(t) \in L^\infty$. From Lemma 4.2, it follows $\dot{\hat{r}}_i(t) \in L^\infty$.

Third, we prove that $\lim_{t\to\infty}(\hat{r}_i(t) - r_i(t)) = 0$ and $\dot{\hat{r}}_i(t) \in L^\infty$ for $i = 1,\ldots,N$. We demonstrate that each agent satisfies $\dot{\hat{r}}_i(t) \in L^\infty$. Let l_i denote the length of the longest directed path for the leader v_0 to the node v_i. Suppose there exists at least one agent v_k such that $\dot{\hat{r}}_k(t)$ is unbounded. Then, there exists a neighbor v_{k_j} of v_k such that $\dot{\hat{r}}_{k_j}$ is unbounded and $l_{k_j} < l_k$. Repeating this analysis for up to l_k steps, it concludes that the reference signal of the leader $\dot{r}(t)$ is unbounded, which contradicts Assumption (A5). Therefore, $\dot{\hat{r}}_i(t) \in L^\infty,\ i = 1,\ldots,N$. Then, we get $m_i(t) \in L^\infty$, $\dot{u}_i(t) \in L^\infty$and $\dot{y}_i(t) \in L^\infty$ and Lemma 4.2 indicates $\lim_{t\to\infty}(\hat{r}_i(t) - r_i(t)) = 0$ and $\dot{\hat{r}}_i(t) \in L^\infty$.

Finally, we demonstrate the tracking convergence and the higher-order properties. From the second and third steps, we get $\lim_{t\to\infty}(y_i(t) - \frac{1}{N_i}\sum_{v_j\in\mathcal{N}_i}y_j(t))^{(k)} = 0$, for any $k = 0,\ldots,n^*, i = 1,\ldots,N$. This together with Lemma 3.6 indicates that $\lim_{t\to\infty}\left(y_i(t) - y_0(t)\right)^{(k)} = 0$ for all $k = 0,\ldots,n^*$ and $i = 1,\ldots,N$. The proof is completed. □

Remark 4.4. Theorem 4.3 addresses the tracking performance in the presence of unknown parameters. If the reference signal $r_0(t)$ meets certain additional conditions, such as being sufficiently rich of order $2\bar{n}$, then the tracking error can further converge to zero exponentially. For more details, please refer to reference [10].

So far, we have established a fully distributed output feedback MRAC scheme, where the adaptive control law for each follower only relies on its local input and output information, and the asymptotic leader-follower output consensus is achieved. Particularly, the proposed adaptive control scheme overcomes the restrictive structural matching conditions, e.g., (2.4) and (2.5), commonly used in the existing distributed MRAC literature. Moreover, the higher-order leader-follower output consensus is achieved without using the persistent excitation condition as shown in Theorem 4.3.

5. Simulation examples. This section presents an example to demonstrate the design procedure and verify Theorem 3.7, Lemma 4.2 and Theorem 4.3. We study the consensus performance of four followers and a virtual leader for the nominal control case and adaptive control case, and their associated communication graph is shown in Fig.1.

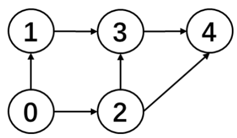

FIG. 1. *Communication graph for nominal control design.*

Simulation system. Consider the following MAS containing four followers modeled as

$$P_i(s)[y_i](t){=}k_{pi}Z_i(s)[u_i](t), t\geq 0, i=1,2,3,4, \tag{5.1}$$

where $P_1(s)=(s{+}1)\left(s-\frac{1}{2}\right), Z_1(s)=s{+}\frac{1}{2}, P_2(s)=\left(s+\frac{3}{2}\right)\left(s-\frac{1}{2}\right)\left(s+\frac{1}{2}\right), Z_2(s)=\left(s+\frac{1}{2}\right)(s+1), P_3(s)=(s-1)(s+2), Z_3(s)=s+\frac{1}{3}, P_4(s)=(s-1)\left(s-\frac{1}{2}\right)(s+2)$, $Z_4(s)=\left(s+\frac{1}{3}\right)\left(s+\frac{1}{4}\right)$, and $k_{p1}=-1/3, k_{p2}=2, k_{p3}=-3, k_{p4}=4$. Note that the followers' models considered in this simulation are unstable and heterogeneous. The leader model is chosen as

$$y_0(t)=W_m(s)\left[r_0\right](t) \tag{5.2}$$

with $W_m(s)=1/P_m(s)=\frac{1}{s+1}$ and $y_0(t)=5\sin(2t)$. Thus, we calculate that $r(t)=10\cos(2t)+5\sin(2t)$.

Nominal control case. When the parameters are known, we utilize distributed MRC law to achieve convergence.

Distributed MRC law specification. Based on (3.1), the distributed MRC law for the MAS (5.1)-(5.2) is designed as

$$u_i(t){=}\theta_{1i}^{*T}\omega_{1i}(t)+\theta_{2i}^{*T}\omega_{2i}(t)+\theta_{20i}^{*}y_i(t)+\theta_{3i}^{*}\omega_{3i}(t), \tag{5.3}$$

where $\omega_{ji}(t), j=1,2,3$, can be derived from (3.2) and (3.3) with $\Lambda_{c1}(s)=s+1, \Lambda_{c2}(s)=s^2+1.5s+0.5, \Lambda_{c3}(s)=s+1, \Lambda_{c4}(s)=s^2+1.5s+0.5$, and $\Psi(s)=s+1.5$. Moreover, by Lemma 3.4 , the matching parameters in (5.3) are calculated as

$$\begin{aligned}
&\theta_{11}^*=0.5,\ \theta_{21}^*=0,\ \theta_{201}^*=4.5,\ \theta_{31}^*=-3,\ \theta_{12}^*=[-53.5,-53.5]^T,\\
&\theta_{22}^*=[-33.625,-13.75]^T,\ \theta_{202}^*=26.25,\ \theta_{32}^*=0.5,\\
&\theta_{13}^*=0.6667,\ \theta_{23}^*=0.6667,\ \theta_{203}^*=0.5,\ \theta_{33}^*=-0.3333,\\
&\theta_{14}^*=[0.4167,0.9167]^T,\ \theta_{24}^*=[0.3750,-0.3750]^T,\ \theta_{204}^*=-0.6250,\ \theta_{34}^*=0.25.
\end{aligned}$$

System responses. The initial outputs of the followers are chosen as $[y_1(0),y_2(0),y_3(0),y_4(0)]^T=[3.5,6,0,8.3]^T$. Fig.2 shows the response of the outputs $y_i(t), i=1,\ldots,4$, of the followers and the trajectories of the derivatives of the leader and followers' output. Fig.2 highlights that the desired output higher order consensus performance is ensured. The simulation results verify the theoretical results.

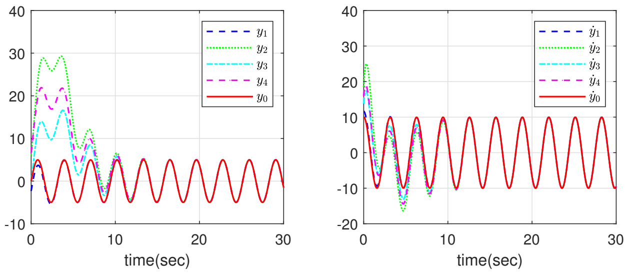

FIG. 2. *Trajectories of the five agents' outputs and derivatives.*

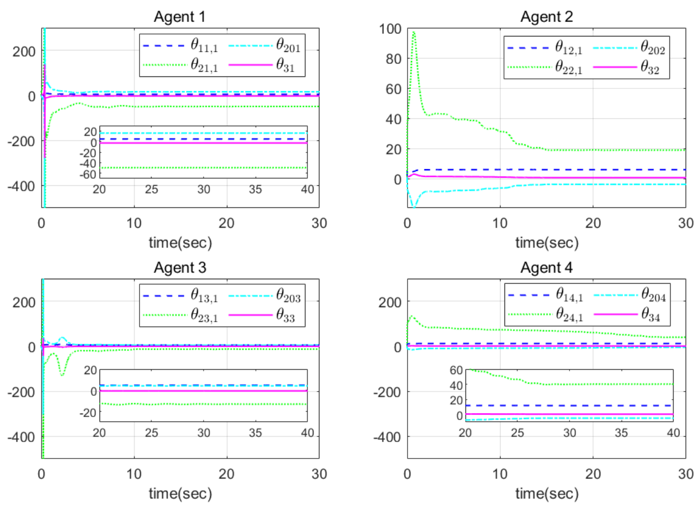

FIG. 3. *Trajectories of the parameter adaptation.*

Adaptive control case. To verify Lemma 4.2 and Theorem 4.3, consider the system (5.1)-(5.2) where the parameters are unknown.

Distributed MRAC law specification. Based on (4.1), the distributed MRAC law for the MAS (5.1)-(5.2) is designed as

$$u_i(t) = \theta_{1i}^T(t)\omega_{1i}(t) + \theta_{2i}^T(t)\omega_{2i}(t) + \theta_{20i}(t)y_i(t) + \theta_{3i}(t)\hat{\omega}_{3i}(t), \tag{5.4}$$

where $\omega_{ji}(t), j = 1, 2$, can be derived from (3.2) with $\Lambda_{c1}(s) = s + 4, \Lambda_{c2}(s) = s^2 + 5s + 6, \Lambda_{c3}(s) = s + 5$, $\Lambda_{c4}(s) = s^2 + 7s + 12$, and $\Psi(s) = s + 1.5$. Moreover, to obtain the adaptive parameters $\theta_{1i}(t), \theta_{2i}(t), \theta_{20i}(t), \theta_{3i}(t)$ in (5.4), first by (4.3), we obtain the estimates of θ_{pi}^* defined in (3.5) with $\Gamma_1 = \Gamma_3 = 10I_{4\times4}, \Gamma_2 = \Gamma_4 = 10I_{6\times6}$, and $\Lambda_{e1}(s) = s^2 + 3s + 2, \Lambda_{e2}(s) = s^3 + 1.833s^2 + s + 0.167, \Lambda_{e3}(s) = s^2 + 1.333s + 0.333, \Lambda_{e4}(s) = s^3 + 1.833s^2 + s + 0.167$, where $\phi_i(t)$, $\epsilon_i(t)$ and $m_i(t)$ can be derived from (3.6), (4.2) and (4.4), respectively. Then, $\theta_{1i}(t), \theta_{2i}(t), \theta_{20i}(t), \theta_{3i}(t)$ can be calculated by (4.6) and (4.7). Next, we specify the signal (4.8) as

$$\hat{\omega}_{31}(t) = \hat{\omega}_{32}(t) = r(t),\ \hat{\omega}_{33}(t) = 1/2(\hat{r}_1(t) + \hat{r}_2(t)),\ \hat{\omega}_{34}(t) = 1/2(\hat{r}_2(t) + \hat{r}_3(t)),$$

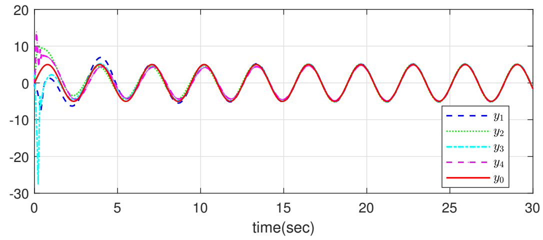

FIG. 4. *Trajectories of the agents' outputs.*

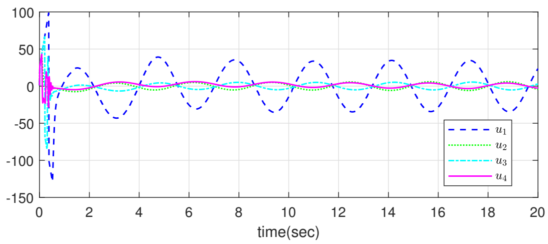

FIG. 5. *Trajectories of the followers' inputs.*

where

$$\hat{r}_j(t) = \theta_{pj}^T(t)s[\phi_j](t) + \frac{s\Lambda_{j,n-1}(s)}{\Lambda_{ej}(s)}[y_j](t) + 1.5y_j(t), j = 1, 2, 3, 4,$$

with $\phi_j(t)$ defined in (3.6) and $\Lambda_{j(n-1)}(s)$ defined below (3.7).

System responses. The initial outputs of the followers are chosen as $[y_1(0), y_2(0), y_3(0), y_4(0)]^T = [-1, 2, 3, 1]^T$. Fig.3 displays the first element of the adaptive parameters $\{\theta_{1i}(t), \theta_{2i}(t), \theta_{20i}(t), \theta_{3i}(t)\}$ in (5.4) and Fig.4 presents the responses of the outputs $y_i(t), i = 1, \ldots, 4$, of the followers. Fig.4 reveals that the desired output consensus performance is ensured. Besides, Fig.5 shows the trajectories of the followers' inputs, and Fig.6 displays the consistency of the estimated virtual reference signal. From Fig.6, Lemma 4.2 is well verified. Fig.7 illustrates the trajectories of the first derivative of the leader and followers' output, highlighting that the higher-order properties in Theorem 4.3 are well supported by the numerical example. Overall, the simulation results have verified the theoretical results for the adaptive control case. Here we provide only numerical examples, while how to apply the proposed method in a real application is currently under investigation.

6. Conclusion. This paper proposes a fully distributed output feedback MRAC method for a general class of linear time-invariant systems with unknown parameters. The developed architecture overcomes the restrictive matching condition commonly used in the existing distributed MRAC methods. Our adaptive control law solely relies on local input and output information and ensures global higher-order leader-follower output consensus. Several simulation results verify the validity of the proposed adaptive control method. Nevertheless, how to solve the issues when the MAS (1)-(2) with uncertain switching topologies by using a distributed output feedback MRAC framework should be further studied.

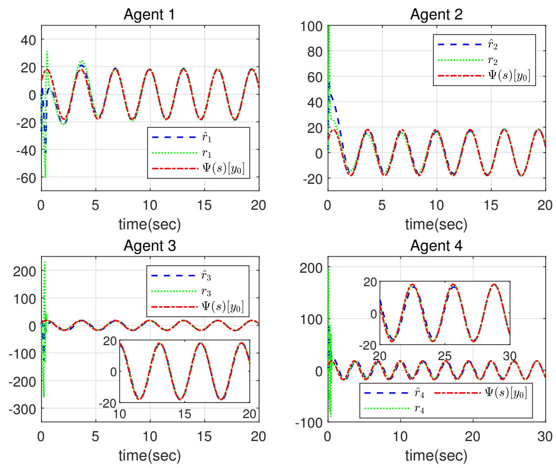

FIG. 6. *Trajectories of the followers' virtual signals.*

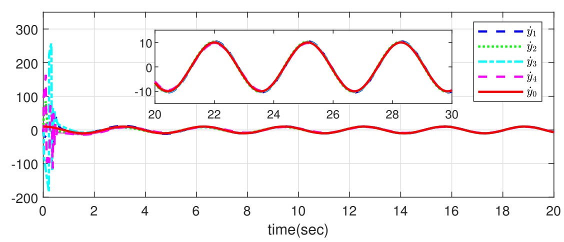

FIG. 7. *Trajectories of the agents' output derivatives.*

Appendix A. Some useful lemmas and theorems. The following lemma establishes a crucial link between the square integrability property of a function and the asymptotic convergence of an associated error signal. Specifically, it states that if a function $f(t)$ has a bounded derivative and the integral $\int_0^\infty f^2(t)dt$ is finite, then $f(t)$ asymptotically approaches zero as $t \to \infty$. This lemma is a specific application of a more general result known as Barbălat's Lemma, which guarantees the convergence of certain types of functions under the given conditions [10].

LEMMA A.1. [37] *If* $\dot{f}(t) \in L^\infty$ *and* $f(t) \in L^2$*, then* $\lim_{t\to\infty} f(t) = 0$.

Now we present some well-known results of traditional indirect MRAC of LTI systems, which are fundamentals in our distributed output feedback MRAC design.

Consider a traditional indirect MRAC system. The control system is

$$P(s)[y](t) = k_p Z(s)[u](t), \tag{A.1}$$

where y is the output, u is the input, $P(s)$ is the pole polynomial with unknown coefficients, $Z(s)$ is the stable zero polynomial with unknown coefficients, and k_p is the unknown high-frequency gain. The reference model is

$$P_m(s)\,[y_m]\,(t) = r(t). \tag{A.2}$$

The indirect MRAC law is

$$u(t) = \theta_1^T \omega_1(t) + \theta_2^T \omega_2(t) + \theta_{20} y(t) + \theta_3 r(t), \tag{A.3}$$

where θ_i, $i = 1, 2, 20, 3$, are designed parameters, $\omega_1(t) = \frac{a(s)}{\Lambda_c(s)}[u](t) \in \mathbb{R}^{n-1}, \omega_2(t) = \frac{a(s)}{\Lambda_c(s)}[y](t) \in \mathbb{R}^{n-1}$ with $a(s) = \left[1, s, \dots, s^{n-2}\right]$ and $\Lambda_c(s)$ being a monic stable polynomial of degree $n-1$.

LEMMA A.2. [37] *There exist constant parameters* $\theta_1^*, \theta_2^*, \theta_{20}^*, \theta_3^*$ *such that*

$$\text{(A.4)}\quad \theta_1^{*T} a(s)P(s) + \left(\theta_2^{*T} a(s) + \theta_{20}^* \Lambda_c(s)\right) Z(s) = \Lambda_c(s)\left(P(s) - \theta_3^* Z(s) P_m(s)\right).$$

THEOREM A.3. [37] *If the parameters* θ_i *in (A.3) are replaced by* θ_i^*, $i = 1, 2, 20, 3$, *satisfying (A.4), then the control law (A.3) ensures that all signals in the closed-loop system are bounded and* $y(t) - y_m(t) = \epsilon_0(t)$ *for some initial condition-related exponentially decaying* $\epsilon_0(t)$.

For the adaptive case, there are two steps to design θ_i, $i = 1, 2, 20, 3$: (i) estimation of the system parameters by an adaptive law like (4.3), and (ii) calculation of the controller parameters using some linear equations like (31). Under some standard assumptions, the indirect MRAC system (A.1)-(A.3) has the following properties. All these properties can be seen in [37]:

THEOREM A.4. [37] *The adaptive control law (A.3) ensures that all signals are bounded and* $y(t) - y_m(t) \in L^2, \lim_{t\to\infty}(y(t) - y_m(t)) = 0$.

Appendix B. Proofs of Lemma 3.2 and Lemma 4.2.

B.1. Proof of Lemma 3.2. Using $\Lambda_{ei}(s)$ defined below (3.6), we can express the agent model (1) of the following form

$$\text{(B.1)}\qquad y_i(t) - \frac{\Lambda_{i(n_i-1)}(s)}{\Lambda_{ei}(s)}[y_i](t) = \theta_{pi}^{*T}\phi_i(t).$$

Then, we have

$$\begin{aligned}
\text{(B.2)}\qquad s[y_i](t) &= \theta_{pi}^{*T} s[\phi_i](t) + \frac{s\Lambda_{i(n_i-1)}(s)}{\Lambda_{ei}(s)}[y_i](t)\\
&= \theta_{pi}^{*T}\left[\frac{s}{\Lambda_{ei}(s)}[u_i](t), \dots, \frac{s^{m_i+1}}{\Lambda_{ei}(s)}[u_i](t)\right.\\
&\qquad \left.\frac{s}{\Lambda_{ei}(s)}[y_i](t), \dots, \frac{s^{n_i}}{\Lambda_{ei}(s)}[y_i](t)\right]^T + \frac{s\Lambda_{i(n_i-1)}(s)}{\Lambda_{ei}(s)}[y_i](t).
\end{aligned}$$

Since the degree of $\Lambda_{ei}(s)$ is n_i, then $\frac{s}{\Lambda_{ei}(s)}[u_i](t), \dots, \frac{s^{m_i+1}}{\Lambda_{ei}(s)}[u_i](t)$ and $\frac{s}{\Lambda_{ei}(s)}[y_i](t)$, $\dots, \frac{s^{n_i-1}}{\Lambda_{ei}(s)}[y_i](t)$ can be expressed by $\phi_i(t)$.

Moreover, we calculate

$$\begin{aligned}
\frac{s^{n_i}}{\Lambda_{ei}(s)}[y_i](t) &= y_i(t) + \frac{s^{n_i} - \Lambda_{ei}(s)}{\Lambda_{ei}(s)}[y_i](t),\\
\frac{s\Lambda_{i(n_i-1)}(s)}{\Lambda_{ei}(s)}[y_i](t) &= \Lambda_{i(n_i-1)}^e y_i(t) + \frac{s\Lambda_{i(n_i-1)}(s) - \Lambda_{i(n_i-1)}^e \Lambda_{ei}(s)}{\Lambda_{ei}(s)}[y_i](t),
\end{aligned}$$

where $\frac{s^{n_i} - \Lambda_{ei}(s)}{\Lambda_{ei}(s)}$, and $\frac{s\Lambda_{i(n_i-1)}(s) - \Lambda_{ei}(s)}{\Lambda_{ei}(s)}$ are strictly proper. This indicates that Lemma 3.2 holds for $j = 1$.

When $1 < j < n^*$, we have

$$s^j[y_i](t) = \theta_{pi}^{*T} s^j[\phi_i](t) + \frac{s^j\Lambda_{i(n_i-1)}(s)}{\Lambda_{ei}(s)}[y_i](t)$$

$$= \theta_{pi}^{*T}\left[\frac{s^j}{\Lambda_{ei}(s)}[u_i](t),\ldots,\frac{s^{m_i+j}}{\Lambda_{ei}(s)}[u_i](t)\ \frac{s^j}{\Lambda_{ei}(s)}[y_i](t),\ldots,\frac{s^{n_i-1+j}}{\Lambda_{ei}(s)}[y_i](t)\right]^T$$

$$+\frac{s^j\Lambda_{i(n_i-1)}(s)}{\Lambda_{ei}(s)}[y_i](t). \tag{B.3}$$

Noting that $j < n^*$, $n_i = m_i + n^*$, the signals $\frac{s^j}{\Lambda_{ei}(s)}[u_i](t),\ldots,\frac{s^{m_i+j}}{\Lambda_{ei}(s)}[u_i](t)$, and $\frac{s^j}{\Lambda_{ei}(s)}[y_i](t),\ldots,\frac{s^{j+(n_i-1-j)}}{\Lambda_{ei}(s)}[y_i](t)$ can be directly obtained. Moreover, through decomposition, one can obtain

$$\frac{s^{n_i+q}}{\Lambda_{ei}(s)} = \sum_{k=0}^{q}\bar{h}_{qk}s^{q-k} + \sum_{k=1}^{n_i-1}\bar{l}_{qk}\frac{s^k}{\Lambda_{ei}(s)}, q = 0,\ldots,j-1,$$

$$\frac{s^j\Lambda_{i(n_i-1)}(s)}{\Lambda_{ei}(s)} = \sum_{k=0}^{j-1}\breve{h}_k s^{j-1-k} + \sum_{k=1}^{n_i-1}\breve{l}_k\frac{s^k}{\Lambda_{ei}(s)}. \tag{B.4}$$

Thereby, $s^j[y_i](t),\ j = 1, 2, \ldots, n^*-1$ can be expressed by $s[y_i](t),\ldots,s^{j-1}[y_i](t)$, θ_{pi}^* in (3.5), $\frac{s^k}{\Lambda_{ei}(s)}[u_i](t), k = 1+m_i,\ldots,j+m_i$, $\phi_i(t)$, and $y_i(t)$.

When $j = n^*$, only the signal $\frac{s^{m_i+j}}{\Lambda_{ei}(s)}[u_i](t)$ needs to be considered. Concretely, $\frac{s^{m_i+j}}{\Lambda_{ei}(s)}[u_i](t) = \frac{s^{n_i}}{\Lambda_{ei}(s)}[u_i](t) = u_i(t) + \frac{s^{n_i}-\Lambda_{ei}(s)}{\Lambda_{ei}(s)}[u_i](t)$ with $\frac{s^{n_i}-\Lambda_{ei}(s)}{\Lambda_{ei}(s)}$ being strictly proper, which indicates the conclusion also holds for $j = n^*$. Thus, the lemma follows. □

B.2. Proof of Lemma 4.2. We first demonstrate that $d_{i1}(t)$ converges to $s[y_i](t)$ by showing that the error term involving $\tilde{\theta}_{pi}(t)$ approaches zero as $t \to \infty$. Using mathematical induction, we extend this result to $d_{ik}(t)$, showing that it converges to $s^k[y_i](t)$ for higher orders. Combining these results, we then establish that the tracking error $\hat{r}_i(t) - r_i(t)$ converges to zero. The detailed proof process is as follows. With (3.8), we define

$$d_{ij}(t) = H_{ij}\left(y_i, \frac{s^{1+m_i}}{\Lambda_{ei}(s)}[u_i],\ldots,\frac{s^{j+m_i}}{\Lambda_{ei}(s)}[u_i], \theta_{pi}, \phi_i\right), \tag{B.5}$$

for $i = 1, ..., N$ and $j = 0, ..., n^*$. Comparing (3.8) and (B.5), we see that $d_{ij}(t)$, $j = 0, ..., n^*$, are the estimates of $y_i(t)$, $s[y_i](t)$, ..., $s^{n^*}[y_i](t)$, respectively. Since $\dot{\theta}_{pi}(t) \in L^\infty$, $\dot{\omega}_{1i}^e(t) \in L^\infty$, $\dot{\omega}_{2i}^e(t) \in L^\infty$, $\dot{u}_i(t) \in L^\infty$and $\dot{y}_i(t) \in L^\infty$, it follows that $\dot{\hat{r}}_i(t) \in L^\infty$. Next, we will prove a stronger conclusion that

$$d_{ij}(t) - s^j[y_i](t) \to 0,\ j = 0,\ldots,n^*. \tag{B.6}$$

We now use mathematical induction to prove (B.6). The proving technique refers to the proof of the higher-order tracking property of MRAC in [38].

Let $\tilde{\theta}_{pi}(t) = \theta_{pi}(t) - \theta_{pi}^*$. When $j = 1$, from (B.1), the signal d_{i1} defined in (B.5) can be expressed by

$$d_{i1}(t) = \theta_{pi}^T(t)s[\phi_i](t) + \frac{s\Lambda_{i(n_i-1)}(s)}{\Lambda_{ei}(s)}[y_i](t). \tag{B.7}$$

Then, by (B.2) and (B.7), we have $d_{i1}(t)-s[y_i](t) = \tilde{\theta}_{pi}^T(t)s[\phi_i](t) = \tilde{\theta}_{pi}^T(t)\dot{\phi}_i(t)$. Noting (4.2) and (B.1), $\epsilon_i(t)$ can be expressed by $\epsilon_i(t) = \theta_{pi}^T(t)\phi_i(t) - \theta_{pi}^{*T}\phi_i(t) = \tilde{\theta}_{pi}^T(t)\phi_i(t)$. Then, the derivative of $\epsilon_i(t)$ is $\dot{\epsilon}_i(t) = \dot{\theta}_{pi}^T(t)\phi(t) + \tilde{\theta}_{pi}^T(t)\dot{\phi}_i(t)$. Noting (4.3), we have $\dot{\theta}_{pi}(t) \in L^\infty$ and thus $\dot{\epsilon}_i(t) \in L^\infty$. Hence, by (4.3), we have $\ddot{\theta}_{pi}(t) \in L^\infty$. Since $\dot{\theta}_{pi}(t) \in L^2$ by Lemma 4.1, then Lemma A.1 indicates that $\lim_{t\to\infty}\dot{\theta}_{pi}(t) = 0$. Thus, to prove that $d_{i1}(t) - s[y_i](t) = \tilde{\theta}_{pi}^T(t)\dot{\phi}_i(t)$ converges to zero, it is sufficient to prove $\lim_{t\to\infty}\dot{\epsilon}_i(t) = 0$. Next, we will prove this property by using the definition of limits, i.e., for any given η, there exists a $T = T(\eta) > 0$ such that $|\dot{\epsilon}_i(t)| < \eta$.

We decompose the signal $\dot{\epsilon}_i(t)$ into two fictitious parts: one being small enough and one converging to zero asymptotically with time going to infinity. First, two fictitious $K(s)$ and $H(s)$ are introduced and defined by

$$K(s) = \frac{a^k}{(s+a)^k}, sH(s) = 1 - K(s), \tag{B.8}$$

where $a > 0$ is an adjustable parameter. Thus, given $K(s)$, the filter $H(s)$ is strictly proper (with relative degree one) and stable, and is specified as

$$H(s) = \frac{1}{s}(1 - K(s)) = \frac{1}{s}\frac{(s+a)^k - a^k}{(s+a)^k}. \tag{B.9}$$

Moreover, from [28], it is known that the impulse response function of $H(s)$ is $h(t) = \mathcal{L}^{-1}[H(s)] = e^{-at}\sum_{i=1}^k \frac{a^{k-i}}{(k-i)!}t^{k-i}$ and the L^1 signal norm of $h(t)$ is

$$\|h(\cdot)\|_1 = \int_0^\infty |h(t)|dt = \frac{k}{a}. \tag{B.10}$$

We choose the filter $K(s)$ and $H(s)$ with $k = 2$. Using (B.8) that $1 = sH(s) + K(s)$, we divide $\dot{\epsilon}_i(t)$ into two terms

$$\begin{aligned} \dot{\epsilon}_i(t) &= s[\tilde{\theta}_{pi}^T\phi_i](t) = H(s)s^2[\tilde{\theta}_{pi}^T\phi_i](t) + sK(s)[\tilde{\theta}_{pi}^T\phi_i](t) \\ &= H(s)s^2[\tilde{\theta}_{pi}^T\phi_i](t) + sK(s)[\epsilon_i](t). \end{aligned} \tag{B.11}$$

By the assumption $m_i(t) \in L^\infty$ and Equations (B.3) and (B.4), we have $\phi_i(t), \dot{\phi}_i(t), \ddot{\phi}_i(t) \in L^\infty$. By Lemma 4.1,we have $\dot{\theta}_{pi}(t), \tilde{\theta}_{pi}(t) \in L^\infty$. Therefore, noting $\ddot{\theta}_{pi}(t) \in L^\infty$, it follows

$$s^2[\tilde{\theta}_{pi}^T\phi_i](t) = [\ddot{\theta}_{pi}^T\phi_i + 2\dot{\theta}_{pi}^T\dot{\phi}_i + \tilde{\theta}_{pi}^T\ddot{\phi}_i](t) \in L^\infty. \tag{B.12}$$

Then, from the above L^1 signal norm expression of $H(s)$, $\|h(\cdot)\|_1 = \frac{2}{a}$, we have

$$\left|H(s)s^2[\tilde{\theta}_{pi}^T\phi_i](t)\right| \le \frac{c_1}{a} \tag{B.13}$$

for any $t \ge 0$ and some constant $c_1 > 0$ independent of $a > 0$. We now consider $sK(s)[\epsilon_i](t)$. Since $\dot{\phi}_i(t) \in L^\infty$ and $m_i(t) \in L^\infty$, then $\dot{\epsilon}_i(t) = \dot{\theta}_{pi}^T(t)\phi_i(t) + \left(\theta_{pi}(t) - \theta_{pi}^*\right)^T\dot{\phi}_i(t) \in L^\infty$. By Lemma 4.1 and $m_i(t) \in L^\infty$, we have $\epsilon_i(t) \in L^2$. Using Lemma A.1, it follows $\lim_{t\to\infty}\epsilon_i(t) = 0$. Therefore, since $sK(s)$ is stable and strictly proper, then, for any finite $a > 0$ in $K(s)$,

$$\lim_{t\to\infty} sK(s)[\epsilon_i](t) = 0. \tag{B.14}$$

For any $\eta > 0$, set $a = a(\eta) \geq \frac{2c_1}{\eta}$ for the filter $H(s)$. Then, it follows that for any $t > 0$,

$$\left|H(s)s^2[\tilde{\theta}_{pi}^T\phi_i](t)\right| \leq \frac{c_1}{a} \leq \frac{\eta}{2}. \tag{B.15}$$

Moreover, by $\lim_{t\to\infty} sK(s)[\epsilon_i](t) = 0$, there exists $T = T(a(\eta), \eta) > 0$, such that for any $t > T$,

$$|sK(s)[\epsilon_i](t)| < \frac{\eta}{2}. \tag{B.16}$$

Therefore, due to (B.15) and (B.16), for any $t > T$

$$|\dot{\epsilon}_i(t)| \leq \left|H(s)s^2[\tilde{\theta}_{pi}^T\phi_i](t)\right| + |sK(s)[\epsilon_i](t)| < \frac{\eta}{2} + \frac{\eta}{2} = \eta, \tag{B.17}$$

which implies $\lim_{t\to\infty} \dot{\epsilon}_i(t) = 0$. So far we have proved that

$$\lim_{t\to\infty} (d_{i1}(t) - s[y_i](t)) = 0.$$

Given that for all $j = 1, \ldots, k-1$, $k \leq n^*$, the following properties hold:

$$\lim_{t\to\infty} \epsilon_{i(k-1)}(t) = 0, \quad \lim_{t\to\infty} \left(d_{ij}(t) - s^j[y_i](t)\right) = 0, \tag{B.18}$$

where $\epsilon_{i(k-1)}(t) = \tilde{\theta}_{pi}^T(t)\left(s^{k-1}[\phi_i](t)\right)$. We have the following analysis.

When $j = k$, by (B.1), we have $s^k[y_i](t) = \theta_{pi}^{*T} s^k[\phi_i](t) + \frac{s^k\Lambda_{i(n_i-1)}(s)}{\Lambda_{ei}(s)}[y_i](t)$. Define

$$P(t) = s^k[\phi_i](t), \; Q(t) = \frac{s^k\Lambda_{i(n_i-1)}(s)}{\Lambda_{ei}(s)}[y_i](t). \tag{B.19}$$

Then,

$$s^k[y_i](t) = \theta_{pi}^{*T} P(t) + Q(t). \tag{B.20}$$

For simplicity of presentation, we denote

$$d_{ik}(t) = \theta_{pi}^T(t)\widehat{P}(t) + \widehat{Q}(t), \tag{B.21}$$

where $\widehat{P}(t)$ and $\widehat{Q}(t)$ are the estimates of $P(t)$ and $Q(t)$, respectively. Using (B.4), $Q(t)$ and $\widehat{Q}(t)$ can be expressed by

$$Q(t) = \sum_{l=0}^{k-1} \breve{h}_l s^l[y_i](t) + \sum_{l=1}^{n_i-1} \breve{l}_l \frac{s^l}{\Lambda_{ei}(s)}[y_i](t), \tag{B.22}$$

$$\widehat{Q}(t) = \sum_{l=0}^{k-1} \breve{h}_l d_{il}(t) + \sum_{l=1}^{n_i-1} \breve{l}_l \frac{s^l}{\Lambda_{ei}(s)}[y_i](t). \tag{B.23}$$

Then, by (B.22), (B.23) and the properties given in (B.18), we have

$$\lim_{t\to\infty} (\widehat{Q}(t) - Q(t)) = \lim_{t\to\infty} \left(\sum_{l=1}^{k-1} \breve{h}_l \left(d_{il} - s^l[y_i](t)\right)\right) = 0. \tag{B.24}$$

Similarly, noting that each element of the vector $s^k[\phi_i](t)$ contains $s^{j-1}[y_i](t),\ j = 1,\dots,k$ and some filtered signals on $y_i(t)$ and $u_i(t)$, then by (B.4), (B.18) and similar analysis for the convergence of $\widehat{Q}(t) - Q(t)$, it follows $\lim_{t\to\infty}(\widehat{P}(t) - P(t)) = 0$. Therefore, by (B.20) and (B.21), we have

$$\begin{aligned} &\lim_{t\to\infty}(d_{ik}(t) - s^k[y_i](t)) = \lim_{t\to\infty}\tilde{\theta}_{pi}^T(t)P(t) + \lim_{t\to\infty}\theta_{pi}^T(t)(\widehat{P}(t) - P(t)) \\ &+ \lim_{t\to\infty}(\widehat{Q}(t) - Q(t)) = \lim_{t\to\infty}\tilde{\theta}_{pi}^T(t)P(t). \end{aligned} \tag{B.25}$$

We next prove that $\lim_{t\to\infty}\tilde{\theta}_{pi}^T(t)P(t) = \lim_{t\to\infty}\tilde{\theta}_{pi}^T(t)\left(s^k[\phi_i](t)\right) = 0$. Consider the signal $\epsilon_{i(k-1)}(t) = \tilde{\theta}_{pi}^T(t)\left(s^{k-1}[\phi_i](t)\right)$. Its derivative is

$$\dot{\epsilon}_{i(k-1)}(t) = \dot{\theta}_{pi}^T(t)s^{k-1}[\phi_i](t) + \tilde{\theta}_{pi}^T(t)s^k[\phi_i](t). \tag{B.26}$$

Since $m_i(t) \in L^\infty$ and $\lim_{t\to\infty}\dot{\theta}_{pi}(t) = 0$, it follows $\lim_{t\to\infty}\dot{\theta}_{pi}^T(t)s^{k-1}[\phi_i](t) = 0$. Hence, by (B.26), to prove $\lim_{t\to\infty}\tilde{\theta}_{pi}^T(t)\left(s^k[\phi_i](t)\right) = 0$, it is sufficient to prove $\lim_{t\to\infty}\dot{\epsilon}_{i(k-1)}(t) = 0$. Similar to (B.11), we express $\dot{\epsilon}_{i(k-1)}(t)$ as

$$\begin{aligned} \dot{\epsilon}_{i(k-1)}(t) &= s[\tilde{\theta}_{pi}^T\left(s^{k-1}[\phi_i]\right)](t) \\ &= H(s)s^2[\tilde{\theta}_{pi}^T\left(s^{k-1}[\phi_i]\right)](t) + sK(s)[\epsilon_{i(k-1)}](t). \end{aligned} \tag{B.27}$$

By the assumption $m_i(t) \in L^\infty$ and Equations (B.3) and (B.4), we have, for $k \le n^*$, $s^k\phi_i(t) \in L^\infty$. When $k = n^*$, by the additional assumption $\dot{u}_i(t), \dot{y}_i(t) \in L^\infty$, we have $s^{k+1}\phi_i(t) \in L^\infty$. Moreover, by Lemma 4.1,we have $\dot{\theta}_{pi}(t), \dot{\tilde{\theta}}_{pi}(t) \in L^\infty$. Therefore, noting $\ddot{\theta}_{pi}(t) \in L^\infty$, it follows

$$s^2[\tilde{\theta}_{pi}^T(t)\left(s^{k-1}[\phi_i]\right)](t) = \left[\ddot{\theta}_{pi}^T s^{k-1}[\phi_i] + 2\dot{\theta}_{pi}^T s^k[\phi_i] + \tilde{\theta}_{pi}^T s^{k+1}[\phi_i]\right](t) \in L^\infty.$$

Then, for $j = k$, similar to (B.13), we have $\left|H(s)s^2\left[\tilde{\theta}_{pi}^T s^{k-1}[\phi_i]\right](t)\right| \le \frac{c_k}{a}$, for some $c_k > 0$ independent of a. Since $sK(s)$ is stable and strictly proper, so that, with $\lim_{t\to\infty}\epsilon_{i(k-1)}(t) = 0$, we have $\lim_{t\to\infty} sK(s)[\epsilon_{i(k-1)}](t) = 0$. Hence, similar to (B.17), by choosing suitable parameter $a > 0$ in $H(s)$ and $K(s)$, it can be shown that for any $\eta > 0$, there exists $T = T(\eta, a) > 0$, such that for any $t > T$, it holds $|\dot{\epsilon}_{i(k-1)}(t)| < \eta$. Therefore, $\lim_{t\to\infty}\dot{\epsilon}_{i(k-1)}(t) = 0$. Then, by $\lim_{t\to\infty}\dot{\tilde{\theta}}_{pi}^T(t)s^{k-1}[\phi_i](t) = 0$ as established above (B.27), and (B.25), we have

$$\lim_{t\to\infty}\epsilon_{ik}(t) = \lim_{t\to\infty}\tilde{\theta}_{pi}^T(t)\left(s^k[\phi_i](t)\right) = 0,\ \lim_{t\to\infty}\left(d_{ik}(t) - s^k[y_i](t)\right) = 0. \tag{B.28}$$

Therefore, by (3.8), (3.9), (4.9), and (B.5), it follows

$$\hat{r}_i(t) - r_i(t) = \sum_{j=0}^{n^*}\psi_j\left(d_{ij}(t) - s^j[y_i](t)\right) \to 0,$$

with ψ_j defined below (3.9). The proof is completed. □

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